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Prospective teachers' attitude towards computer algebra systems (CAS) and their choice of using CAS in solving problems of systems of differential equations

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ABSTRACT: In this study, the authors investigate prospective mathematics teachers' (PMTs) attitudes towards using computer algebra systems (CAS) in the learning of systems of differential equations. Systems of differential equations typically arise in the physical and natural sciences as a way to describe two or more simultaneous rates of change. PMTs with sufficient knowledge of systems of differential equations can not only build their lessons at the level of modern requirements, but also contribute to the teaching of other disciplines of the school course, such as physics, chemistry, biology, computer science and computer engineering. Therefore, PMTs should have a broad scientific and practical knowledge of systems of differential equations. Adopting the survey research design, the participants of the study were 57 PMTs enrolled in the *Differential Equations*, and *Using ICT in Mathematics (Maple CAS)* courses at a university in Kazakhstan. In view of the students' responses to survey questions, the authors confirm that the PMTs' attitudes towards using CAS in solving problems of systems of differential equations were generally positive.

INTRODUCTION

The course of differential equations plays an essential role in the fundamental preparation of a future teacher in terms of the formation of a student's scientific worldview, a certain level of mathematical culture, a certain level of methodological culture, especially in such components as understanding the essence of the applied and practical orientation of teaching mathematics, mastering the method of mathematical modelling and the ability to carry out interdisciplinary connections in teaching [1].

Taking into account modern requirements for specialists in the field of mathematics, differential equations course content should contain topics rich in modern methods. The course on differential equations, on the one hand, is very abstract and has its specifics, terminology, its models, often quite subtle. Studying this course, students are often confused and do not understand why all this is necessary for a future teacher. On the other hand, the course on differential equations is one of the most advantageous for the future teacher's awareness of the essence of mathematics, its applied orientation and its educational significance.

The concept of a differential equation is one of the basic mathematical concepts. Differential equations are obtained when the various states of the phenomenon or process under study can be described analytically by the dependence between some parameters of this process and their derivatives or differentials.

The authors note that for the study of objects whose mathematical description leads to differential equations, knowledge of the laws of physics, chemistry, biology, ecology, etc, i.e. those branches of knowledge with which the nature of the object under study is associated, is of great and sometimes of the highest importance [2].

The computer as a means of teaching opens up a favourable opportunity for the revival of Comenius's didactics at a particular technical level [3]. Didactic ideas and techniques embedded in the educational material and control programmes in computer training will work productively for each student, considering the required time and the student's attributes. Thus, the point is to rely on didactic principles and modern technology in order to create a learning system that allows the teacher to implement the learning process with high efficiency.

One of the great benefits of a computer algebra system (CAS) in a teaching environment is that it allows students to experiment and explore, while at the same time removing the cognitive load associated with messy algebraic manipulations [4][5]. The use of CAS as the main means of information technology for solving mathematical problems is a necessary part of the training of a PMT.

The most effective study of computer algebra systems focused on a particular section of mathematics is learning these programmes within that section. With such an organisation of training, students should be able to simultaneously obtain the necessary theoretical knowledge and learn to implement the studied mathematical methods with the help of information technology.

MATLAB CAS was used as a tool to enhance students' understanding in topics, such as electric circuit theory, wireless communication theory, and the motion of a mass vibrating up and down at the end of a spring [6-8]. According to the findings of the study by Stewart et al, Maple is a reasonably friendly worksheet environment combining text, commands and pictures all in one place, while MATLAB is a programming language, where the commands, pictures and numerical output all appear in different places [4]. The Wolfram Mathematica system was applied to solve descriptive geometry problems for the improvement of geometric and graphical training of students [9].

In their studies, Grünwald et al selected two models; namely, *a model for water quality control in Lake Schwerin* and *a model for fishing in Lake Sternberg,* to involve first-year engineering students in solving real-life problems from their first year of study, increase their motivation for the subject of mathematics and encourage them to take responsibility for environmental issues [10].

The traditional teaching methods of differential equations involve the mastery of symbolic and algebraic rules for solving problems in Kazakhstani universities, particularly at *Khoja Akhmet Yassawi* International Kazakh-Turkish University and at the South Kazakhstan State Pedagogical University.

According to the comparative analysis results of the study by Yessengabylov et al, a low assessment of the importance of using ICT in teaching mathematics is a hindering factor, while a high level of knowledge and skills in working with ICT increases the use of ICT for that activity [11]. However, even a high level of individual skills does not lead to substantially more productive use of ICT, and the development of a wide range of teachers' skills at a sufficient level is a prerequisite for being more productive in the use of ICT to teach mathematics.

Systems of differential equations are one of the advanced topics in differential equations. Some research studies indicated that systems of differential equations are not an easy concept to understand because it involves the concept of linear algebra, such as eigenvalues and eigenvectors [12][13].

In their studies, Goodchild et al explored eight students' development of critical stances in an inquiry-oriented differential equations course for prospective secondary mathematics teachers [14]. In another study, Nerona examined the effectiveness of collaborative learning strategies on engineering students' achievement in various engineering courses, including differential equations [15].

In their earlier article, Kalimbetov and Kulakhmetova discussed the use of the Dana-Picard and Steiner's *low-level* commands of Maple CAS as a tool to help the student to achieve a better understanding of the differential equations concept [16][17]. In their study, Bidaibekov et al described and discussed the methodological aspects of teaching differential equations as one of the applied courses based on humanistic values [18].

SOLVING SYSTEMS OF DIFFERENTIAL EQUATIONS USING MAPLE

In this section, the authors consider examples of using Maple for solving systems of differential equations.

1. Solve the following system of a differential equation.

$$\begin{cases} \dfrac{dx}{dt} = -\dfrac{1}{4}x + 2y \\ \dfrac{dy}{dt} = -8x - \dfrac{1}{4}y \end{cases}$$

In matrix form, the system is equivalent to the system $X' = \begin{pmatrix} -\frac{1}{4} & 2 \\ -8 & -\frac{1}{4} \end{pmatrix} X$. The eigenvalues and corresponding eigenvectors $A = \begin{pmatrix} -\frac{1}{4} & 2 \\ -8 & -\frac{1}{4} \end{pmatrix}$ are found to be $\lambda_{1,2} = -\frac{1}{4} \pm 4i$ and $\mathbf{v}_{1,2} = \begin{pmatrix} 0 \\ 2 \end{pmatrix} \pm \begin{pmatrix} 1 \\ 0 \end{pmatrix} i$ with eigenvectors.

Below, the authors include examples of using Maple to see and check solutions step by step (Figure 1 to Figure 4).

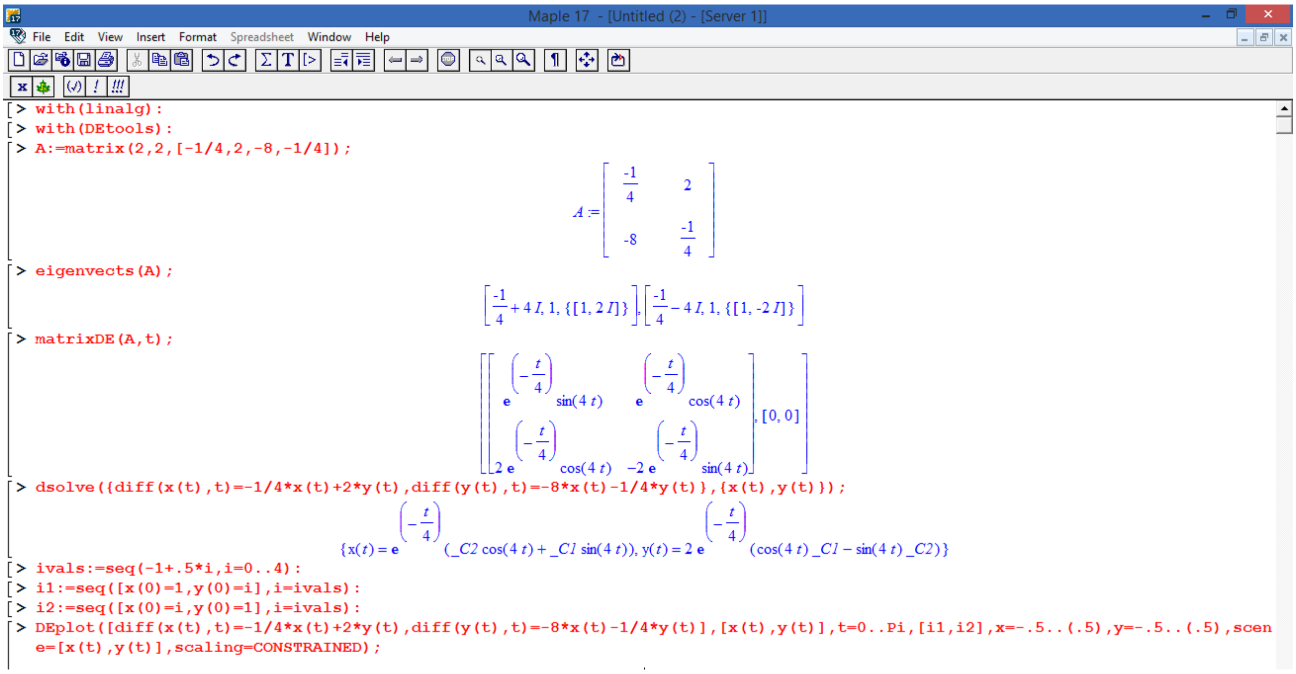

Figure 1: A screen capture of the solution of the systems of differential equation $\begin{cases} \dfrac{dx}{dt} = -\dfrac{1}{4}x + 2y \\ \dfrac{dy}{dt} = -8x - \dfrac{1}{4}y \end{cases}$ in a Maple CAS.

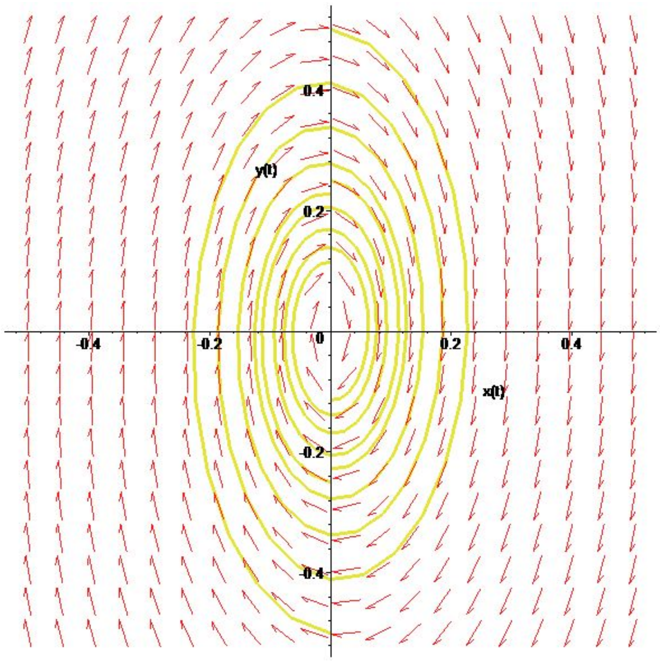

Figure 2: A screen capture of the graphically presented solution of the systems of differential equations:

$$\begin{cases} \dfrac{dx}{dt} = -\dfrac{1}{4}x + 2y \\ \dfrac{dy}{dt} = -8x - \dfrac{1}{4}y \end{cases}.$$

2. Solve the initial-value problem.

$$\mathbf{X}' = \begin{pmatrix} 1 & -1 \\ 10 & -1 \end{pmatrix}\mathbf{X} - \begin{pmatrix} t\cos 3t \\ t\sin t + t\cos 3t \end{pmatrix},\ \mathbf{X}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}.$$

The solution is as follows: The corresponding homogeneous system is $\mathbf{X}_h' = \begin{pmatrix} 1 & -1 \\ 10 & -1 \end{pmatrix} \mathbf{X}_h$ .The eigenvalue and corresponding eigenvectors of $\mathbf{A} = \begin{pmatrix} 1 & -1 \\ 10 & -1 \end{pmatrix}$ are $\lambda_{1,2} = \pm 3i$ and $\mathbf{v}_{1,2} = \begin{pmatrix} 1 \\ 10 \end{pmatrix} \pm \begin{pmatrix} -3 \\ 0 \end{pmatrix} i$, respectively. A fundamental matrix is: $\Phi = \begin{pmatrix} \sin 3t & \cos 3t \\ \sin 3t - 3\cos 3t & \cos 3t + 3\sin 3t \end{pmatrix}$. One can now compute using the formula: $\mathbf{X}_p = \Phi \int \Phi^{-1} \mathbf{F} dt$.

A general solution of the non-homogeneous system is then:

$\mathbf{X} = \mathbf{X}_h + \mathbf{X}_p = \Phi C + \Phi \int \Phi^{-1} \mathbf{F} dt = \mathbf{X} = \mathbf{X}_h + \mathbf{X}_p = \Phi(C + \int \Phi^{-1} \mathbf{F} dt) = \Phi \int \Phi^{-1} \mathbf{F} dt$, where one has incorporated the constant vector $\mathbf{C}$ into the indefinite integral $\int \Phi^{-1} \mathbf{F} dt$.

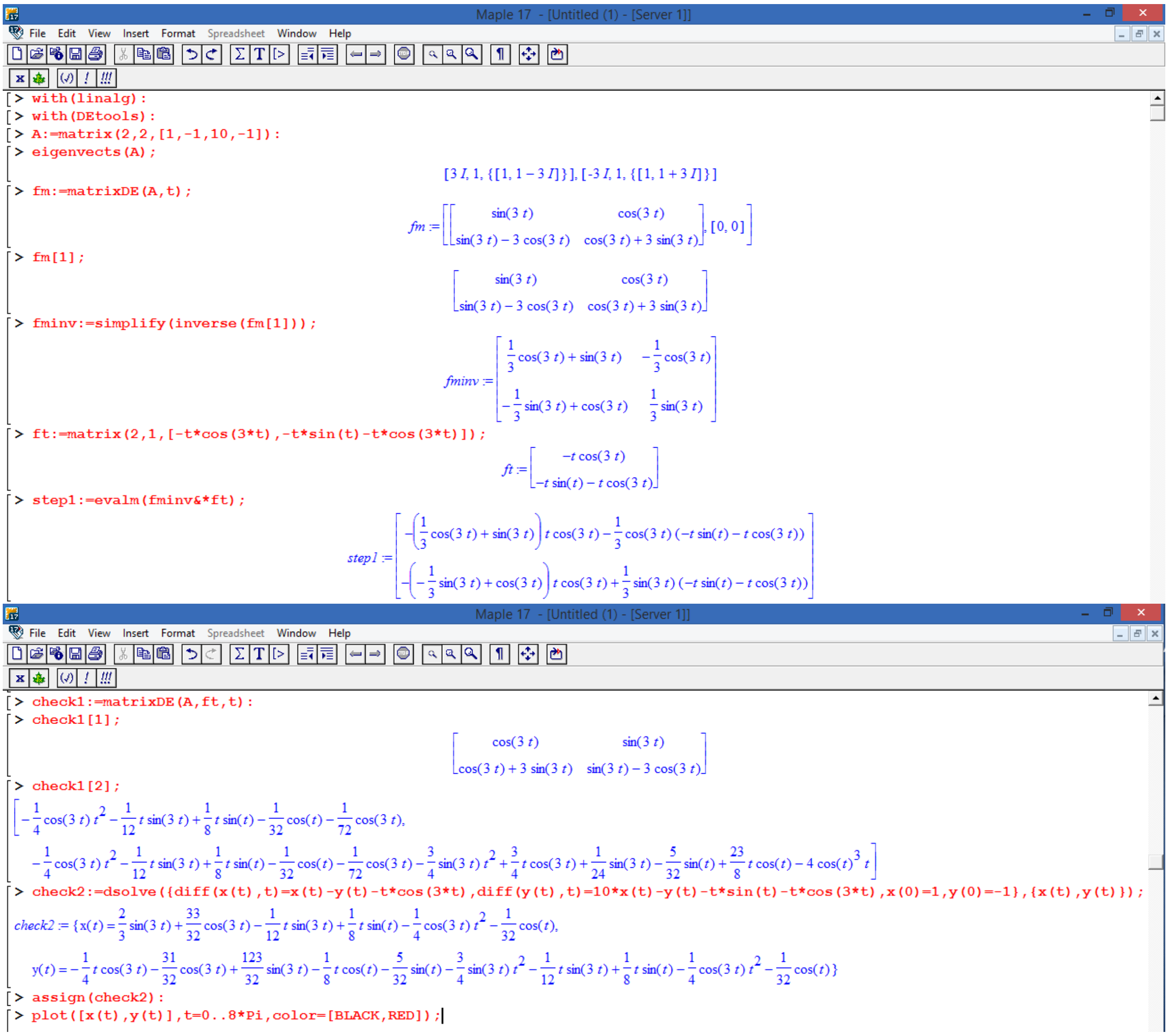

Figure 3: Screen captures of the solution of the systems of differential equations $\mathbf{X}' = \begin{pmatrix} 1 & -1 \\ 10 & -1 \end{pmatrix} \mathbf{X} - \begin{pmatrix} t\cos 3t \\ t\sin t + t\cos 3t \end{pmatrix}$, $\mathbf{X}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ in a Maple CAS.

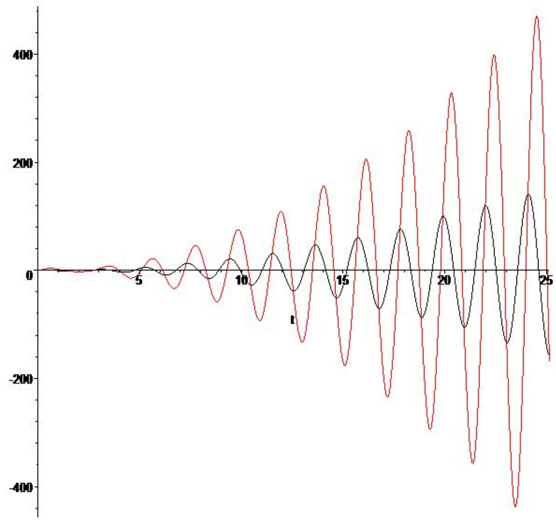

Figure 4: A screen capture of the graphically presented solution of the systems of differential equations.

$$\mathbf{X}' = \begin{pmatrix} 1 & -1 \\ 10 & -1 \end{pmatrix}\mathbf{X} - \begin{pmatrix} t\cos 3t \\ t\sin t + t\cos 3t \end{pmatrix},\ \mathbf{X}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$ in a Maple CAS.

METHODOLOGY

In Kazakhstan, both graduates of secondary mathematics teacher education programmes and mathematics programmes have a chance to be mathematics teachers in secondary schools. Therefore, participants of this study were senior students from secondary mathematics teacher education programmes (n = 24) and senior students from mathematics programmes (n = 33) at IKTU.

The study participants after the course completion on differential equations, and with experience in using ICT in mathematics (Maple CAS) were asked about their attitude towards using Maple when solving problems of systems of differential equations. As indicated in Table 1, the survey included seven, five-point Likert scale questions (from 5 - very high/strongly agree to 1 - strongly disagree/very low).

Table 1: Likert scale survey.

	Survey questions	5	4	3	2	1
Q1	Evaluate the importance of Maple CAS for computational tasks.					
Q2	Evaluate the importance of Maple CAS for solving problems of systems of differential equations.					
Q3	Assess the degree of applicability of the CAS in your future educational activities.					
Q4	How important is knowledge of the CAS in your professional activity?					
Q5	Evaluate the practical significance of the lesson.					
Q6	Assess the timeliness of studying the CAS.					
Q7	To what extent does studying the CAS increase interest in your future profession?					

RESULTS AND DISCUSSIONS

The survey results are presented in Table 2 (combined results from a to h).

Table 2: Statistical analysis of the PMTs’ survey responses.

a) Statistics

		Q1	Q2	Q3	Q4	Q5	Q6	Q7
N	Valid	57	57	57	57	57	57	57
	Missing	0	0	0	0	0	0	0
Mode		4.00	4.00	4.00	3.00	4.00	4.00	4.00
Range		2.00	2.00	3.00	2.00	2.00	2.00	2.00
Minimum		3.00	3.00	2.00	2.00	3.00	3.00	3.00
Maximum		5.00	5.00	5.00	4.00	5.00	5.00	5.00

b) Question 1

		Frequency	Percent	Valid percent	Cumulative percent
Valid	Neutral	6	10.5	10.5	10.5
	Agree	30	52.6	52.6	63.2
	Strongly agree	21	36.8	36.8	100.0
	Total	57	100.0	100.0	

c) Question 2

		Frequency	Percent	Valid percent	Cumulative percent
Valid	Neutral	22	38.6	38.6	38.6
	Agree	31	54.4	54.4	93.0
	Strongly agree	4	7.0	7.0	100.0
	Total	57	100.0	100.0	

d) Question 3

		Frequency	Percent	Valid percent	Cumulative percent
Valid	Disagree	3	5.3	5.3	5.3
	Neutral	18	31.6	31.6	36.8
	Agree	35	61.4	61.4	98.2
	Strongly agree	1	1.8	1.8	100.0
	Total	57	100.0	100.0	

e) Question 4

		Frequency	Percent	Valid percent	Cumulative percent
Valid	Neutral	5	8.8	8.8	8.8
	Agree	40	70.2	70.2	78.9
	Strongly agree	12	21.1	21.1	100.0
	Total	57	100.0	100.0	

f) Question 5

		Frequency	Percent	Valid percent	Cumulative percent
Valid	Neutral	11	19.3	19.3	19.3
	Agree	41	71.9	71.9	91.2
	Strongly agree	5	8.8	8.8	100.0
	Total	57	100.0	100.0	

g) Question 6

		Frequency	Percent	Valid percent	Cumulative percent
Valid	Neutral	4	7.0	7.0	7.0
	Agree	42	73.7	73.7	80.7
	Strongly agree	11	19.3	19.3	100.0
	Total	57	100.0	100.0	

h) Question 7

		Frequency	Percent	Valid percent	Cumulative percent
Valid	Neutral	2	3.5	3.5	3.5
	Agree	40	70.2	70.2	73.7
	Strongly agree	15	26.3	26.3	100.0
	Total	57	100.0	100.0	

When analysing the results of the survey as shown in Table 2, the authors have drawn the following conclusions:

- The absolute majority of the students note the great importance of Maple CAS for simplifying computational calculations and solving mathematical problems.
- Knowledge of Maple CAS will help students in both academic and professional activities.
- Most students point to the high practical significance of the lesson.
- The study of Maple CAS stimulates further professional activity of students.

CONCLUSIONS

The standard differential equation courses do not foster the qualitative meanings of basic concepts, such as the rate of change, integral, existence and uniqueness, etc [19]. A point was made that the PMT needs a good conceptual understanding of pre-requisite courses, such as linear algebra to be able to correctly interpret the results of solving problems of systems of differential equations using Maple CAS.

As a final word, learning systems of differential equations in a CAS environment helps to improve PMTs' beliefs about the relevance of CAS in their future profession.

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